Bridging the Gap

Between GCSE and A level Mathematics

This booklet will remind you how to:

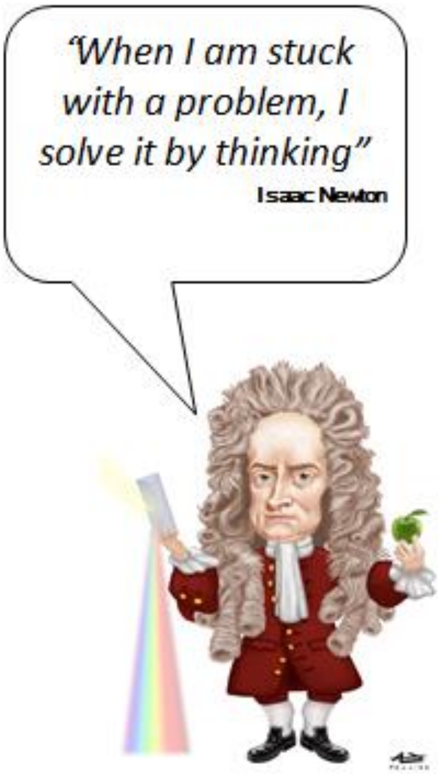
- Manipulate algebraic expressions and evaluate formulae
- Solve linear equations and graph linear equations
- Find the midpoint and find the length of a line segment
- Solve simultaneous equations and deal with inequalities.

Please complete this booklet so that you are prepared for a test on this material in your first lesson.

A key skill to be successful at A level Mathematics is to be an independent learner. You have covered all the material in this booklet in your GCSE course, if you can't remember how to approach some of the questions here are some suggestions of how to get started:

- Look at your GCSE maths notes/ textbooks/ revision guides
- Look on the Web (websites such as http://www.bbc.co.uk/schools/gcsebitesize/maths/)
- Ask a friend, sibling or parent/ guardian.

The answers are at the back of the booklet. They're placed there as a learning tool, so that you can check your work to ensure that you're on track. Use them wisely!



Good luck and welcome to Maths at JCoSS

1.1 Index laws

- **You can use the laws of indices to simplify powers of the same base.**
  - $a^m \times a^n = a^{m+n}$
  - $a^m \div a^n = a^{m-n}$
  - $(a^m)^n = a^{mn}$
  - $(ab)^n = a^n b^n$

Notation

x^5 — This is the **base**. This is the **index**, **power** or **exponent**.

Example 1

Simplify these expressions:

a $x^2 \times x^5$ **b** $2r^2 \times 3r^3$ **c** $\frac{b^7}{b^4}$ **d** $6x^5 \div 3x^3$ **e** $(a^3)^2 \times 2a^2$ **f** $(3x^2)^3 \div x^4$

a $x^2 \times x^5 = x^{2+5} = x^7$

Use the rule $a^m \times a^n = a^{m+n}$ to simplify the index.

b $2r^2 \times 3r^3 = 2 \times 3 \times r^2 \times r^3$
$= 6 \times r^{2+3} = 6r^5$

Rewrite the expression with the numbers together and the r terms together.
$2 \times 3 = 6$
$r^2 \times r^3 = r^{2+3}$

c $\frac{b^7}{b^4} = b^{7-4} = b^3$

Use the rule $a^m \div a^n = a^{m-n}$ to simplify the index.

d $6x^5 \div 3x^3 = \frac{6}{3} \times \frac{x^5}{x^3}$
$= 2 \times x^2 = 2x^2$

$x^5 \div x^3 = x^{5-3} = x^2$

e $(a^3)^2 \times 2a^2 = a^6 \times 2a^2$
$= 2 \times a^6 \times a^2 = 2a^8$

Use the rule $(a^m)^n = a^{mn}$ to simplify the index.

$a^6 \times a^2 = a^{6+2} = a^8$

f $\frac{(3x^2)^3}{x^4} = 3^3 \times \frac{(x^2)^3}{x^4}$
$= 27 \times \frac{x^6}{x^4} = 27x^2$

Use the rule $(ab)^n = a^n b^n$ to simplify the numerator.
$(x^2)^3 = x^{2 \times 3} = x^6$

$\frac{x^6}{x^4} = x^{6-4} = x^2$

JCoSS

Example 3

Simplify these expressions:

a $\dfrac{x^7 + x^4}{x^3}$ **b** $\dfrac{3x^2 - 6x^5}{2x}$ **c** $\dfrac{20x^7 + 15x^3}{5x^2}$

a $\dfrac{x^7 + x^4}{x^3} = \dfrac{x^7}{x^3} + \dfrac{x^4}{x^3}$

Divide each term of the numerator by x^3.

$= x^{7-3} + x^{4-3} = x^4 + x$

x^1 is the same as x.

b $\dfrac{3x^2 - 6x^5}{2x} = \dfrac{3x^2}{2x} - \dfrac{6x^5}{2x}$

Divide each term of the numerator by $2x$.

$= \dfrac{3}{2}x^{2-1} - 3x^{5-1} = \dfrac{3x}{2} - 3x^4$

Simplify each fraction:

$\dfrac{3x^2}{2x} = \dfrac{3}{2} \times \dfrac{x^2}{x} = \dfrac{3}{2} \times x^{2-1}$

$-\dfrac{6x^5}{2x} = -\dfrac{6}{2} \times \dfrac{x^5}{x} = -3 \times x^{5-1}$

c $\dfrac{20x^7 + 15x^3}{5x^2} = \dfrac{20x^7}{5x^2} + \dfrac{15x^3}{5x^2}$

Divide each term of the numerator by $5x^2$.

$= 4x^{7-2} + 3x^{3-2} = 4x^5 + 3x$

Exercise 1A

1 Simplify these expressions:

a $x^3 \times x^4$ **b** $2x^3 \times 3x^2$ **c** $\dfrac{k^3}{k^2}$

d $\dfrac{4p^3}{2p}$ **e** $\dfrac{3x^3}{3x^2}$ **f** $(y^2)^5$

g $10x^5 \div 2x^3$ **h** $(p^3)^2 \div p^4$ **i** $(2a^3)^2 \div 2a^3$

j $8p^4 \div 4p^3$ **k** $2a^4 \times 3a^5$ **l** $\dfrac{21a^3b^7}{7ab^4}$

m $9x^2 \times 3(x^2)^3$ **n** $3x^3 \times 2x^2 \times 4x^6$ **o** $7a^4 \times (3a^4)^2$

p $(4y^3)^3 \div 2y^3$ **q** $2a^3 \div 3a^2 \times 6a^5$ **r** $3a^4 \times 2a^5 \times a^3$

2 Expand and simplify if possible:

a $9(x - 2)$ **b** $x(x + 9)$ **c** $-3y(4 - 3y)$

d $x(y + 5)$ **e** $-x(3x + 5)$ **f** $-5x(4x + 1)$

g $(4x + 5)x$ **h** $-3y(5 - 2y^2)$ **i** $-2x(5x - 4)$

j $(3x - 5)x^2$ **k** $3(x + 2) + (x - 7)$ **l** $5x - 6 - (3x - 2)$

m $4(c + 3d^2) - 3(2c + d^2)$ **n** $(r^2 + 3t^2 + 9) - (2r^2 + 3t^2 - 4)$

o $x(3x^2 - 2x + 5)$ **p** $7y^2(2 - 5y + 3y^2)$ **q** $-2y^2(5 - 7y + 3y^2)$

r $7(x - 2) + 3(x + 4) - 6(x - 2)$ **s** $5x - 3(4 - 2x) + 6$

t $3x^2 - x(3 - 4x) + 7$ **u** $4x(x + 3) - 2x(3x - 7)$ **v** $3x^2(2x + 1) - 5x^2(3x - 4)$

3 Simplify these fractions:

a $\dfrac{6x^4 + 10x^6}{2x}$ **b** $\dfrac{3x^5 - x^7}{x}$ **c** $\dfrac{2x^4 - 4x^2}{4x}$

d $\dfrac{8x^3 + 5x}{2x}$ **e** $\dfrac{7x^7 + 5x^2}{5x}$ **f** $\dfrac{9x^5 - 5x^3}{3x}$

Exercise 1B

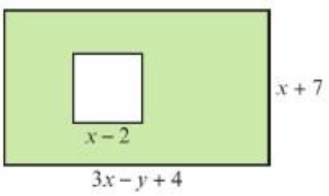
1 Expand and simplify if possible:

a $(x+4)(x+7)$	**b** $(x-3)(x+2)$	**c** $(x-2)^2$
d $(x-y)(2x+3)$	**e** $(x+3y)(4x-y)$	**f** $(2x-4y)(3x+y)$
g $(2x-3)(x-4)$	**h** $(3x+2y)^2$	**i** $(2x+8y)(2x+3)$
j $(x+5)(2x+3y-5)$	**k** $(x-1)(3x-4y-5)$	**l** $(x-4y)(2x+y+5)$
m $(x+2y-1)(x+3)$	**n** $(2x+2y+3)(x+6)$	**o** $(4-y)(4y-x+3)$
p $(4y+5)(3x-y+2)$	**q** $(5y-2x+3)(x-4)$	**r** $(4y-x-2)(5-y)$

2 Expand and simplify if possible:

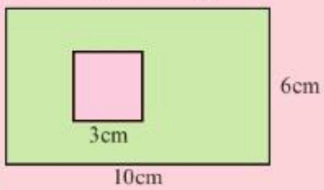
a $5(x+1)(x-4)$	**b** $7(x-2)(2x+5)$	**c** $3(x-3)(x-3)$
d $x(x-y)(x+y)$	**e** $x(2x+y)(3x+4)$	**f** $y(x-5)(x+1)$
g $y(3x-2y)(4x+2)$	**h** $y(7-x)(2x-5)$	**i** $x(2x+y)(5x-2)$
j $x(x+2)(x+3y-4)$	**k** $y(2x+y-1)(x+5)$	**l** $y(3x+2y-3)(2x+1)$
m $x(2x+3)(x+y-5)$	**n** $2x(3x-1)(4x-y-3)$	**o** $3x(x-2y)(2x+3y+5)$
p $(x+3)(x+2)(x+1)$	**q** $(x+2)(x-4)(x+3)$	**r** $(x+3)(x-1)(x-5)$
s $(x-5)(x-4)(x-3)$	**t** $(2x+1)(x-2)(x+1)$	**u** $(2x+3)(3x-1)(x+2)$
v $(3x-2)(2x+1)(3x-2)$	**w** $(x+y)(x-y)(x-1)$	**x** $(2x-3y)^3$

(P) 3 The diagram shows a rectangle with a square cut out.
The rectangle has length $3x-y+4$ and width $x+7$.
The square has length $x-2$.
Find an expanded and simplified expression for the shaded area.



Problem-solving

Use the same strategy as you would use if the lengths were given as numbers:



(P) 4 A cuboid has dimensions $x+2$ cm, $2x-1$ cm and $2x+3$ cm.
Show that the volume of the cuboid is $4x^3+12x^2+5x-6$ cm^3.

(E/P) 5 Given that $(2x+5y)(3x-y)(2x+y)=ax^3+bx^2y+cxy^2+dy^3$, where a, b, c and d are constants, find the values of a, b, c and d. **(2 marks)**

Challenge

Expand and simplify $(x+y)^4$.

Links You can use the binomial expansion to expand expressions like $(x+y)^4$ quickly.
→ Section 8.3

Exercise 1C

1 Factorise these expressions completely:

a $4x + 8$ **b** $6x - 24$ **c** $20x + 15$
d $2x^2 + 4$ **e** $4x^2 + 20$ **f** $6x^2 - 18x$
g $x^2 - 7x$ **h** $2x^2 + 4x$ **i** $3x^2 - x$
j $6x^2 - 2x$ **k** $10y^2 - 5y$ **l** $35x^2 - 28x$
m $x^2 + 2x$ **n** $3y^2 + 2y$ **o** $4x^2 + 12x$
p $5y^2 - 20y$ **q** $9xy^2 + 12x^2y$ **r** $6ab - 2ab^2$
s $5x^2 - 25xy$ **t** $12x^2y + 8xy^2$ **u** $15y - 20yz^2$
v $12x^2 - 30$ **w** $xy^2 - x^2y$ **x** $12y^2 - 4yx$

2 Factorise:

a $x^2 + 4x$ **b** $2x^2 + 6x$ **c** $x^2 + 11x + 24$
d $x^2 + 8x + 12$ **e** $x^2 + 3x - 40$ **f** $x^2 - 8x + 12$
g $x^2 + 5x + 6$ **h** $x^2 - 2x - 24$ **i** $x^2 - 3x - 10$
j $x^2 + x - 20$ **k** $2x^2 + 5x + 2$ **l** $3x^2 + 10x - 8$
m $5x^2 - 16x + 3$ **n** $6x^2 - 8x - 8$
o $2x^2 + 7x - 15$ **p** $2x^4 + 14x^2 + 24$
q $x^2 - 4$ **r** $x^2 - 49$
s $4x^2 - 25$ **t** $9x^2 - 25y^2$ **u** $36x^2 - 4$
v $2x^2 - 50$ **w** $6x^2 - 10x + 4$ **x** $15x^2 + 42x - 9$

Hint For part **n**, take 2 out as a common factor first. For part **p**, let $y = x^2$.

3 Factorise completely:

a $x^3 + 2x$ **b** $x^3 - x^2 + x$ **c** $x^3 - 5x$
d $x^3 - 9x$ **e** $x^3 - x^2 - 12x$ **f** $x^3 + 11x^2 + 30x$
g $x^3 - 7x^2 + 6x$ **h** $x^3 - 64x$ **i** $2x^3 - 5x^2 - 3x$
j $2x^3 + 13x^2 + 15x$ **k** $x^3 - 4x$ **l** $3x^3 + 27x^2 + 60x$

E/P 4 Factorise completely $x^4 - y^4$. **(2 marks)**

Problem-solving
Watch out for terms that can be written as a function of a function: $x^4 = (x^2)^2$

E 5 Factorise completely $6x^3 + 7x^2 - 5x$. **(2 marks)**

1.4 Negative and fractional indices

Indices can be negative numbers or fractions.

$x^{\frac{1}{2}} \times x^{\frac{1}{2}} = x^{\frac{1}{2}+\frac{1}{2}} = x^1 = x,$

similarly $\underbrace{x^{\frac{1}{n}} \times x^{\frac{1}{n}} \times \ldots \times x^{\frac{1}{n}}}_{n \text{ terms}} = x^{\frac{1}{n}+\frac{1}{n}+\ldots+\frac{1}{n}} = x^1 = x$

Notation **Rational** numbers are those that can be written as $\frac{a}{b}$ where a and b are integers.

- **You can use the laws of indices with any rational power.**
  - $a^{\frac{1}{m}} = \sqrt[m]{a}$
  - $a^{\frac{n}{m}} = \sqrt[m]{a^n}$
  - $a^{-m} = \frac{1}{a^m}$
  - $a^0 = 1$

Notation $a^{\frac{1}{2}} = \sqrt{a}$ is the positive square root of a.
For example $9^{\frac{1}{2}} = \sqrt{9} = 3$ but $9^{\frac{1}{2}} \neq -3$.

1 Simplify:

a $x^3 \div x^{-2}$ **b** $x^5 \div x^7$ **c** $x^{\frac{3}{2}} \times x^{\frac{5}{2}}$

d $(x^2)^{\frac{3}{2}}$ **e** $(x^3)^{\frac{5}{3}}$ **f** $3x^{0.5} \times 4x^{-0.5}$

g $9x^{\frac{2}{3}} \div 3x^{\frac{1}{6}}$ **h** $5x^{\frac{7}{5}} \div x^{\frac{2}{5}}$ **i** $3x^4 \times 2x^{-5}$

j $\sqrt{x} \times \sqrt[3]{x}$ **k** $(\sqrt{x})^3 \times (\sqrt[3]{x})^4$ **l** $\frac{(\sqrt[3]{x})^2}{\sqrt{x}}$

2 Evaluate:

a $25^{\frac{1}{2}}$ **b** $81^{\frac{3}{2}}$ **c** $27^{\frac{1}{3}}$

d 4^{-2} **e** $9^{-\frac{1}{2}}$ **f** $(-5)^{-3}$

g $\left(\frac{3}{4}\right)^0$ **h** $1296^{\frac{3}{4}}$ **i** $\left(\frac{25}{16}\right)^{\frac{3}{2}}$

j $\left(\frac{27}{8}\right)^{\frac{2}{3}}$ **k** $\left(\frac{6}{5}\right)^{-1}$ **l** $\left(\frac{343}{512}\right)^{-\frac{2}{3}}$

3 Simplify:

a $(64x^{10})^{\frac{1}{2}}$ **b** $\frac{5x^3 - 2x^2}{x^5}$ **c** $(125x^{12})^{\frac{1}{3}}$ **d** $\frac{x + 4x^3}{x^3}$

e $\frac{2x + x^2}{x^4}$ **f** $\left(\frac{4}{9}x^4\right)^{\frac{3}{2}}$ **g** $\frac{9x^2 - 15x^5}{3x^3}$ **h** $\frac{5x + 3x^2}{15x^3}$

E **4** **a** Find the value of $81^{\frac{1}{4}}$. **(1 mark)**

b Simplify $x(2x^{-\frac{1}{3}})^4$. **(2 marks)**

E **5** Given that $y = \frac{1}{8}x^3$ express each of the following in the form kx^n, where k and n are constants.

a $y^{\frac{1}{3}}$ **(2 marks)**

b $\frac{1}{2}y^{-2}$ **(2 marks)**

JCoSS

1.5 Surds

If n is an integer that is **not** a square number, then any multiple of $\sqrt{n}$ is called a surd.
Examples of surds are $\sqrt{2}$, $\sqrt{19}$ and $5\sqrt{2}$.

Surds are examples of **irrational numbers**.
The decimal expansion of a surd is never-ending and never repeats, for example $\sqrt{2} = 1.414213562\ldots$

Notation Irrational numbers cannot be written in the form $\frac{a}{b}$ where a and b are integers.
Surds are examples of **irrational numbers**.

You can use surds to write exact answers to calculations.

- **You can manipulate surds using these rules:**
  - $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
  - $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$

Exercise 1E

1 Do not use your calculator for this exercise. Simplify:

a $\sqrt{28}$ **b** $\sqrt{72}$ **c** $\sqrt{50}$

d $\sqrt{32}$ **e** $\sqrt{90}$ **f** $\dfrac{\sqrt{12}}{2}$

g $\dfrac{\sqrt{27}}{3}$ **h** $\sqrt{20} + \sqrt{80}$ **i** $\sqrt{200} + \sqrt{18} - \sqrt{72}$

j $\sqrt{175} + \sqrt{63} + 2\sqrt{28}$ **k** $\sqrt{28} - 2\sqrt{63} + \sqrt{7}$ **l** $\sqrt{80} - 2\sqrt{20} + 3\sqrt{45}$

m $3\sqrt{80} - 2\sqrt{20} + 5\sqrt{45}$ **n** $\dfrac{\sqrt{44}}{\sqrt{11}}$ **o** $\sqrt{12} + 3\sqrt{48} + \sqrt{75}$

2 Expand and simplify if possible:

a $\sqrt{3}(2 + \sqrt{3})$ **b** $\sqrt{5}(3 - \sqrt{3})$ **c** $\sqrt{2}(4 - \sqrt{5})$

d $(2 - \sqrt{2})(3 + \sqrt{5})$ **e** $(2 - \sqrt{3})(3 - \sqrt{7})$ **f** $(4 + \sqrt{5})(2 + \sqrt{5})$

g $(5 - \sqrt{3})(1 - \sqrt{3})$ **h** $(4 + \sqrt{3})(2 - \sqrt{3})$ **i** $(7 - \sqrt{11})(2 + \sqrt{11})$

E 3 Simplify $\sqrt{75} - \sqrt{12}$ giving your answer in the form $a\sqrt{3}$, where a is an integer. **(2 marks)**

JCoSS

1.6 Rationalising denominators

If a fraction has a surd in the denominator, it is sometimes useful to **rearrange** it so that the denominator is a **rational** number. This is called rationalising the denominator.

- **The rules to rationalise denominators are:**
  - **For fractions in the form $\frac{1}{\sqrt{a}}$, multiply the numerator and denominator by $\sqrt{a}$.**
  - **For fractions in the form $\frac{1}{a+\sqrt{b}}$, multiply the numerator and denominator by $a-\sqrt{b}$.**
  - **For fractions in the form $\frac{1}{a-\sqrt{b}}$, multiply the numerator and denominator by $a+\sqrt{b}$.**

Exercise 1F

1 Simplify:

a $\frac{1}{\sqrt{5}}$ **b** $\frac{1}{\sqrt{11}}$ **c** $\frac{1}{\sqrt{2}}$ **d** $\frac{\sqrt{3}}{\sqrt{15}}$

e $\frac{\sqrt{12}}{\sqrt{48}}$ **f** $\frac{\sqrt{5}}{\sqrt{80}}$ **g** $\frac{\sqrt{12}}{\sqrt{156}}$ **h** $\frac{\sqrt{7}}{\sqrt{63}}$

2 Rationalise the denominators and simplify:

a $\frac{1}{1+\sqrt{3}}$ **b** $\frac{1}{2+\sqrt{5}}$ **c** $\frac{1}{3-\sqrt{7}}$ **d** $\frac{4}{3-\sqrt{5}}$ **e** $\frac{1}{\sqrt{5}-\sqrt{3}}$

f $\frac{3-\sqrt{2}}{4-\sqrt{5}}$ **g** $\frac{5}{2+\sqrt{5}}$ **h** $\frac{5\sqrt{2}}{\sqrt{8}-\sqrt{7}}$ **i** $\frac{11}{3+\sqrt{11}}$ **j** $\frac{\sqrt{3}-\sqrt{7}}{\sqrt{3}+\sqrt{7}}$

k $\frac{\sqrt{17}-\sqrt{11}}{\sqrt{17}+\sqrt{11}}$ **l** $\frac{\sqrt{41}+\sqrt{29}}{\sqrt{41}-\sqrt{29}}$ **m** $\frac{\sqrt{2}-\sqrt{3}}{\sqrt{3}-\sqrt{2}}$

3 Rationalise the denominators and simplify:

a $\frac{1}{(3-\sqrt{2})^2}$ **b** $\frac{1}{(2+\sqrt{5})^2}$ **c** $\frac{4}{(3-\sqrt{2})^2}$

d $\frac{3}{(5+\sqrt{2})^2}$ **e** $\frac{1}{(5+\sqrt{2})(3-\sqrt{2})}$ **f** $\frac{2}{(5-\sqrt{3})(2+\sqrt{3})}$

P **4** Simplify $\frac{3-2\sqrt{5}}{\sqrt{5}-1}$ giving your answer in the form $p+q\sqrt{5}$, where p and q are rational numbers. **(4 marks)**

Problem-solving

You can check that your answer is in the correct form by writing down the values of p and q and checking that they are rational numbers.

Mixed exercise 1

1 Simplify:

a $y^3 \times y^5$ **b** $3x^2 \times 2x^5$ **c** $(4x^2)^3 \div 2x^5$ **d** $4b^2 \times 3b^3 \times b^4$

2 Expand and simplify if possible:

a $(x + 3)(x - 5)$ **b** $(2x - 7)(3x + 1)$ **c** $(2x + 5)(3x - y + 2)$

3 Expand and simplify if possible:

a $x(x + 4)(x - 1)$ **b** $(x + 2)(x - 3)(x + 7)$ **c** $(2x + 3)(x - 2)(3x - 1)$

4 Expand the brackets:

a $3(5y + 4)$ **b** $5x^2(3 - 5x + 2x^2)$ **c** $5x(2x + 3) - 2x(1 - 3x)$ **d** $3x^2(1 + 3x) - 2x(3x - 2)$

5 Factorise these expressions completely:

a $3x^2 + 4x$ **b** $4y^2 + 10y$ **c** $x^2 + xy + xy^2$ **d** $8xy^2 + 10x^2y$

6 Factorise:

a $x^2 + 3x + 2$ **b** $3x^2 + 6x$ **c** $x^2 - 2x - 35$ **d** $2x^2 - x - 3$

e $5x^2 - 13x - 6$ **f** $6 - 5x - x^2$

7 Factorise:

a $2x^3 + 6x$ **b** $x^3 - 36x$ **c** $2x^3 + 7x^2 - 15x$

8 Simplify:

a $9x^3 \div 3x^{-3}$ **b** $(4^{\frac{3}{2}})^{\frac{1}{3}}$ **c** $3x^{-2} \times 2x^4$ **d** $3x^{\frac{1}{3}} \div 6x^{\frac{2}{3}}$

9 Evaluate:

a $\left(\frac{8}{27}\right)^{\frac{2}{3}}$ **b** $\left(\frac{225}{289}\right)^{\frac{3}{2}}$

10 Simplify:

a $\frac{3}{\sqrt{63}}$ **b** $\sqrt{20} + 2\sqrt{45} - \sqrt{80}$

11 **a** Find the value of $35x^2 + 2x - 48$ when $x = 25$.

b By factorising the expression, show that your answer to part **a** can be written as the product of two prime factors.

12 Expand and simplify if possible:

a $\sqrt{2}(3+\sqrt{5})$ **b** $(2-\sqrt{5})(5+\sqrt{3})$ **c** $(6-\sqrt{2})(4-\sqrt{7})$

13 Rationalise the denominator and simplify:

a $\dfrac{1}{\sqrt{3}}$ **b** $\dfrac{1}{\sqrt{2}-1}$ **c** $\dfrac{3}{\sqrt{3}-2}$ **d** $\dfrac{\sqrt{23}-\sqrt{37}}{\sqrt{23}+\sqrt{37}}$ **e** $\dfrac{1}{(2+\sqrt{3})^2}$ **f** $\dfrac{1}{(4-\sqrt{7})^2}$

14 **a** Given that $x^3 - x^2 - 17x - 15 = (x+3)(x^2 + bx + c)$, where b and c are constants, work out the values of b and c.

b Hence, fully factorise $x^3 - x^2 - 17x - 15$.

E 15 Given that $y = \frac{1}{64}x^3$ express each of the following in the form kx^n, where k and n are constants.

a $y^{\frac{1}{3}}$ **(1 mark)**

b $4y^{-1}$ **(1 mark)**

E/P 16 Show that $\dfrac{5}{\sqrt{75}-\sqrt{50}}$ can be written in the form $\sqrt{a}+\sqrt{b}$, where a and b are integers. **(5 marks)**

E 17 Expand and simplify $(\sqrt{11}-5)(5-\sqrt{11})$. **(2 marks)**

E 18 Factorise completely $x - 64x^3$. **(3 marks)**

E/P 19 Express 27^{2x+1} in the form 3^y, stating y in terms of x. **(2 marks)**

E/P 20 Solve the equation $8 + x\sqrt{12} = \dfrac{8x}{\sqrt{3}}$

Give your answer in the form $a\sqrt{b}$ where a and b are integers. **(4 marks)**

P 21 A rectangle has a length of $(1+\sqrt{3})$ cm and area of $\sqrt{12}$ cm^2.
Calculate the width of the rectangle in cm.
Express your answer in the form $a + b\sqrt{3}$, where a and b are integers to be found.

E 22 Show that $\dfrac{(2-\sqrt{x})^2}{\sqrt{x}}$ can be written as $4x^{-\frac{1}{2}} - 4 + x^{\frac{1}{2}}$. **(2 marks)**

E/P 23 Given that $243\sqrt{3} = 3^a$, find the value of a. **(3 marks)**

E/P 24 Given that $\dfrac{4x^3 + x^{\frac{3}{2}}}{\sqrt{x}}$ can be written in the form $4x^a + x^b$, write down the value of a and the value of b. **(2 marks)**

Challenge

a Simplify $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})$.

b Hence show that $\dfrac{1}{\sqrt{1}+\sqrt{2}} + \dfrac{1}{\sqrt{2}+\sqrt{3}} + \dfrac{1}{\sqrt{3}+\sqrt{4}} + \ldots + \dfrac{1}{\sqrt{24}+\sqrt{25}} = 4$

2.1 Solving quadratic equations

A quadratic equation can be written in the form $ax^2 + bx + c = 0$, where a, b and c are real constants, and $a \neq 0$. Quadratic equations can have one, two, or no real solutions.

- **To solve a quadratic equation by factorising:**
  - **Write the equation in the form $ax^2 + bx + c = 0$**
  - **Factorise the left-hand side**
  - **Set each factor equal to zero and solve to find the value(s) of x**

Notation The solutions to an equation are sometimes called the **roots** of the equation.

Exercise 2A

1 Solve the following equations using factorisation:

a $x^2 + 3x + 2 = 0$ b $x^2 + 5x + 4 = 0$ c $x^2 + 7x + 10 = 0$ d $x^2 - x - 6 = 0$

e $x^2 - 8x + 15 = 0$ f $x^2 - 9x + 20 = 0$ g $x^2 - 5x - 6 = 0$ h $x^2 - 4x - 12 = 0$

2 Solve the following equations using factorisation:

a $x^2 = 4x$ b $x^2 = 25x$ c $3x^2 = 6x$ d $5x^2 = 30x$

e $2x^2 + 7x + 3 = 0$ f $6x^2 - 7x - 3 = 0$ g $6x^2 - 5x - 6 = 0$ h $4x^2 - 16x + 15 = 0$

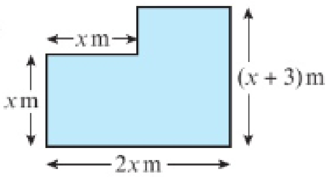
3 Solve the following equations:

a $3x^2 + 5x = 2$ b $(2x - 3)^2 = 9$ c $(x - 7)^2 = 36$ d $2x^2 = 8$ e $3x^2 = 5$

f $(x - 3)^2 = 13$ g $(3x - 1)^2 = 11$ h $5x^2 - 10x^2 = -7 + x + x^2$

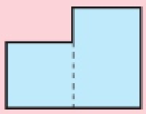
i $6x^2 - 7 = 11x$ j $4x^2 + 17x = 6x - 2x^2$

4 This shape has an area of $44\,\text{m}^2$.
Find the value of x.



Problem-solving

Divide the shape into two sections:



5 Solve the equation $5x + 3 = \sqrt{3x + 7}$.

Exercise 2B

1 Solve the following equations using the quadratic formula.
Give your answers exactly, leaving them in surd form where necessary.

a $x^2 + 3x + 1 = 0$ **b** $x^2 - 3x - 2 = 0$ **c** $x^2 + 6x + 6 = 0$ **d** $x^2 - 5x - 2 = 0$
e $3x^2 + 10x - 2 = 0$ **f** $4x^2 - 4x - 1 = 0$ **g** $4x^2 - 7x = 2$ **h** $11x^2 + 2x - 7 = 0$

2 Solve the following equations using the quadratic formula.
Give your answers to three significant figures.

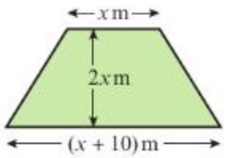
a $x^2 + 4x + 2 = 0$ **b** $x^2 - 8x + 1 = 0$ **c** $x^2 + 11x - 9 = 0$ **d** $x^2 - 7x - 17 = 0$
e $5x^2 + 9x - 1 = 0$ **f** $2x^2 - 3x - 18 = 0$ **g** $3x^2 + 8 = 16x$ **h** $2x^2 + 11x = 5x^2 - 18$

3 For each of the equations below, choose a suitable method and find all of the solutions.
Where necessary, give your answers to three significant figures.

a $x^2 + 8x + 12 = 0$ **b** $x^2 + 9x - 11 = 0$
c $x^2 - 9x - 1 = 0$ **d** $2x^2 + 5x + 2 = 0$
e $(2x + 8)^2 = 100$ **f** $6x^2 + 6 = 12x$
g $2x^2 - 11 = 7x$ **h** $x = \sqrt{8x - 15}$

Hint You can use any method you are confident with to solve these equations.

(P) 4 This trapezium has an area of $50\,\text{m}^2$.
Show that the height of the trapezium is equal to $5(\sqrt{5} - 1)\,\text{m}$.



Problem-solving

Height must be positive. You will have to discard the negative solution of your quadratic equation.

Challenge

Given that x is positive, solve the equation
$$\frac{1}{x} + \frac{1}{x+2} = \frac{28}{195}$$

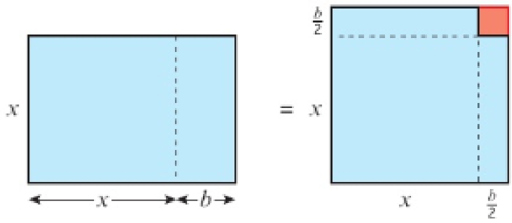
Hint Write the equation in the form $ax^2 + bx + c = 0$ before using the quadratic formula or factorising.

2.2 Completing the square

It is frequently useful to rewrite quadratic expressions by **completing the square:**

- $x^2 + bx = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$

You can draw a diagram of this process when x and b are positive:



The original rectangle has been rearranged into the shape of a square with a smaller square missing. The two areas shaded blue are the same.

$$x^2 + bx = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$$

Exercise 2C

Hint In question **3d**, write the expression as $-4x^2 - 16x + 10$ then take a factor of -4 out of the first two terms to get $-4(x^2 + 4x) + 10$.

1 Complete the square for the expressions:

a $x^2 + 4x$ **b** $x^2 - 6x$ **c** $x^2 - 16x$ **d** $x^2 + x$ **e** $x^2 - 14$

2 Complete the square for the expressions:

a $2x^2 + 16x$ **b** $3x^2 - 24x$ **c** $5x^2 + 20x$ **d** $2x^2 - 5x$ **e** $8x - 2x^2$

3 Write each of these expressions in the form $p(x + q)^2 + r$, where p, q and r are constants to be found:

a $2x^2 + 8x + 1$ **b** $5x^2 - 15x + 3$ **c** $3x^2 + 2x - 1$ **d** $10 - 16x - 4x^2$ **e** $2x - 8x^2 + 10$

E **4** Given that $x^2 + 3x + 6 = (x + a)^2 + b$, find the values of the constants a and b. **(2 marks)**

E **5** Write $2 + 0.8x - 0.04x^2$ in the form $A - B(x + C)^2$, where A, B and C are constants to be determined. **(3 marks)**

JCoSS

Exercise 2D

1 Solve these quadratic equations by completing the square. Leave your answers in surd form.

a $x^2 + 6x + 1 = 0$ **b** $x^2 + 12x + 3 = 0$ **c** $x^2 + 4x - 2 = 0$ **d** $x^2 - 10x = 5$

2 Solve these quadratic equations by completing the square. Leave your answers in surd form.

a $2x^2 + 6x - 3 = 0$ **b** $5x^2 + 8x - 2 = 0$ **c** $4x^2 - x - 8 = 0$ **d** $15 - 6x - 2x^2 = 0$

3 $x^2 - 14x + 1 = (x + p)^2 + q$, where p and q are constants.

a Find the values of p and q. **(2 marks)**

b Using your answer to part **a**, or otherwise, show that the solutions to the equation $x^2 - 14x + 1 = 0$ can be written in the form $r \pm s\sqrt{3}$, where r and s are constants to be found. **(2 marks)**

4 By completing the square, show that the solutions to the equation $x^2 + 2bx + c = 0$ are given by the formula $x = -b \pm \sqrt{b^2 - c}$. **(4 marks)**

Problem-solving

Follow the same steps as you would if the coefficients were numbers.

Challenge

a Show that the solutions to the equation $ax^2 + 2bx + c = 0$ are given by $x = -\frac{b}{a} \pm \sqrt{\frac{b^2 - ac}{a^2}}$.

b Hence, or otherwise, show that the solutions to the equation $ax^2 + bx + c = 0$ can be written as $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Hint Start by dividing the whole equation by a.

Links You can use this method to prove the quadratic formula. → Section 7.4

3.1 Linear simultaneous equations

Linear simultaneous equations in two unknowns have **one set of values** that will make a pair of equations true at the same time.

The solution to this pair of simultaneous equations is $x = 5$, $y = 2$

$x + 3y = 11$ (1) $5 + 3(2) = 5 + 6 = 11$ ✓

$4x - 5y = 10$ (2) $4(5) - 5(2) = 20 - 10 = 10$ ✓

- **Linear simultaneous equations can be solved using elimination or substitution.**

Exercise 3A

1 Solve these simultaneous equations by elimination:

a $2x - y = 6$
$4x + 3y = 22$

b $7x + 3y = 16$
$2x + 9y = 29$

c $5x + 2y = 6$
$3x - 10y = 26$

d $2x - y = 12$
$6x + 2y = 21$

e $3x - 2y = -6$
$6x + 3y = 2$

f $3x + 8y = 33$
$6x = 3 + 5y$

2 Solve these simultaneous equations by substitution:

a $x + 3y = 11$
$4x - 7y = 6$

b $4x - 3y = 40$
$2x + y = 5$

c $3x - y = 7$
$10x + 3y = -2$

d $2y = 2x - 3$
$3y = x - 1$

3 Solve these simultaneous equations:

a $3x - 2y + 5 = 0$
$5(x + y) = 6(x + 1)$

b $\frac{x - 2y}{3} = 4$
$2x + 3y + 4 = 0$

c $3y = 5(x - 2)$
$3(x - 1) + y + 4 = 0$

Hint First rearrange both equations into the same form e.g. $ax + by = c$.

4 $3x + ky = 8$
$x - 2ky = 5$
are simultaneous equations where k is a constant.

a Show that $x = 3$. **(3 marks)**

b Given that $y = \frac{1}{2}$ determine the value of k. **(1 mark)**

Problem-solving k is a constant, so it has the same value in both equations.

5 $2x - py = 5$
$4x + 5y + q = 0$
are simultaneous equations where p and q are constants.
The solution to this pair of simultaneous equations is $x = q$, $y = -1$.
Find the value of p and the value of q. **(5 marks)**

3.2 Quadratic simultaneous equations

You need to be able to solve simultaneous equations where one equation is linear and one is quadratic.

To solve simultaneous equations involving one linear equation and one quadratic equation, you need to use a substitution method from the linear equation into the quadratic equation.

- **Simultaneous equations with one linear and one quadratic equation can have up to two pairs of solutions. You need to make sure the solutions are paired correctly.**

The solutions to this pair of simultaneous equations are $x = 4$, $y = -3$ and $x = 5.5$, $y = -1.5$.

$x - y = 7$ (1) — $4 - (-3) = 7$ ✓ and $5.5 - (-1.5) = 7$ ✓

$y^2 + xy + 2x = 5$ (2) — $(-3)^2 + (4)(-3) + 2(4) = 9 - 12 + 8 = 5$ ✓ and $(-1.5)^2 + (5.5)(-1.5) + 2(5.5) = 2.25 - 8.25 + 11 = 5$ ✓

Exercise 3B

1 Solve the simultaneous equations:

a $x + y = 11$, $xy = 30$

b $2x + y = 1$, $x^2 + y^2 = 1$

c $y = 3x$, $2y^2 - xy = 15$

d $3a + b = 8$, $3a^2 + b^2 = 28$

e $2u + v = 7$, $uv = 6$

f $3x + 2y = 7$, $x^2 + y = 8$

2 Solve the simultaneous equations:

a $2x + 2y = 7$, $x^2 - 4y^2 = 8$

b $x + y = 9$, $x^2 - 3xy + 2y^2 = 0$

c $5y - 4x = 1$, $x^2 - y^2 + 5x = 41$

3 Solve the simultaneous equations, giving your answers in their simplest surd form:

a $x - y = 6$, $xy = 4$

b $2x + 3y = 13$, $x^2 + y^2 = 78$

Watch out Use brackets when you are substituting an expression into an equation.

4 Solve the simultaneous equations:

$x + y = 3$

$x^2 - 3y = 1$ **(6 marks)**

5 **a** By eliminating y from the equations

$y = 2 - 4x$

$3x^2 + xy + 11 = 0$

show that $x^2 - 2x - 11 = 0$. **(2 marks)**

b Hence, or otherwise, solve the simultaneous equations

$y = 2 - 4x$

$3x^2 + xy + 11 = 0$

giving your answers in the form $a \pm b\sqrt{3}$, where a and b are integers. **(5 marks)**

6 One pair of solutions for the simultaneous equations

$y = kx - 5$

$4x^2 - xy = 6$

is $(1, p)$ where k and p are constants.

a Find the values of k and p.

b Find the second pair of solutions for the simultaneous equations.

Problem-solving If $(1, p)$ is a solution, then $x = 1$, $y = p$ satisfies both equations.

Challenge

$y - x = k$

$x^2 + y^2 = 4$

Given that the simultaneous equations have exactly one pair of solutions, show that

$k = \pm 2\sqrt{2}$

3.4 Linear inequalities

You can solve linear inequalities using similar methods to those for solving linear equations.

- **The solution of an inequality is the set of all real numbers x that make the inequality true.**

Exercise 3D

1 Find the set of values of x for which:

a $2x - 3 < 5$

b $5x + 4 \geqslant 39$

c $6x - 3 > 2x + 7$

d $5x + 6 \leqslant -12 - x$

e $15 - x > 4$

f $21 - 2x > 8 + 3x$

g $1 + x < 25 + 3x$

h $7x - 7 < 7 - 7x$

i $5 - 0.5x \geqslant 1$

j $5x + 4 > 12 - 2x$

2 Find the set of values of x for which:

a $2(x - 3) \geqslant 0$

b $8(1 - x) > x - 1$

c $3(x + 7) \leqslant 8 - x$

d $2(x - 3) - (x + 12) < 0$

e $1 + 11(2 - x) < 10(x - 4)$

f $2(x - 5) \geqslant 3(4 - x)$

g $12x - 3(x - 3) < 45$

h $x - 2(5 + 2x) < 11$

i $x(x - 4) \geqslant x^2 + 2$

j $x(5 - x) \geqslant 3 + x - x^2$

k $3x + 2x(x - 3) \leqslant 2(5 + x^2)$

l $x(2x - 5) \leqslant \dfrac{4x(x + 3)}{2} - 9$

3 Use set notation to describe the set of values of x for which:

a $3(x - 2) > x - 4$ and $4x + 12 > 2x + 17$

b $2x - 5 < x - 1$ and $7(x + 1) > 23 - x$

c $2x - 3 > 2$ and $3(x + 2) < 12 + x$

d $15 - x < 2(11 - x)$ and $5(3x - 1) > 12x + 19$

e $3x + 8 \leqslant 20$ and $2(3x - 7) \geqslant x + 6$

f $5x + 3 < 9$ or $5(2x + 1) > 27$

g $4(3x + 7) \leqslant 20$ or $2(3x - 5) \geqslant \dfrac{7 - 6x}{2}$

Challenge

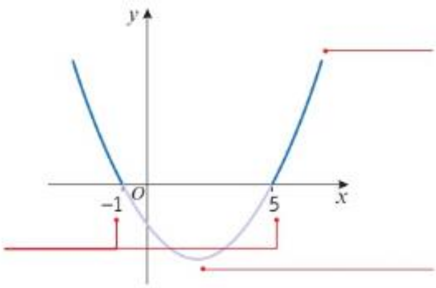
$A = \{x : 3x + 5 > 2\}$ $\quad B = \left\{x : \dfrac{x}{2} + 1 \leqslant 3\right\}$ $\quad C = \{x : 11 < 2x - 1\}$

Given that $A \cap (B \cup C) = \{x : p < x \leqslant q\} \cup \{x : x > r\}$, find the values of p, q and r.

3.5 Quadratic inequalities

- **To solve a quadratic inequality:**
  - **Rearrange so that the right-hand side of the inequality is 0**
  - **Solve the corresponding quadratic equation to find the critical values**
  - **Sketch the graph of the quadratic function**
  - **Use your sketch to find the required set of values.**

The sketch shows the graph of $f(x) = x^2 - 4x - 5$
$= (x + 1)(x - 5)$



The solutions to $f(x) = 0$ are $x = -1$ and $x = 5$. These are called the critical values.

The solutions to the quadratic inequality $x^2 - 4x - 5 > 0$ are the x-values when the curve is **above** the x-axis (the darker part of the curve). This is when $x < -1$ or $x > 5$. In set notation the solution is $\{x : x < -1\} \cup \{x : x > 5\}$.

The solutions to the quadratic inequality $x^2 - 4x - 5 < 0$ are the x-values when the curve is **below** the x-axis (the lighter part of the curve). This is when $x > -1$ and $x < 5$ or $-1 < x < 5$. In set notation the solution is $\{x : -1 < x < 5\}$.

Exercise 3E

1 Find the set of values of x for which:

a $x^2 - 11x + 24 < 0$ **b** $12 - x - x^2 > 0$ **c** $x^2 - 3x - 10 > 0$

d $x^2 + 7x + 12 \geqslant 0$ **e** $7 + 13x - 2x^2 > 0$ **f** $10 + x - 2x^2 < 0$

g $4x^2 - 8x + 3 \leqslant 0$ **h** $-2 + 7x - 3x^2 < 0$ **i** $x^2 - 9 < 0$

j $6x^2 + 11x - 10 > 0$ **k** $x^2 - 5x > 0$ **l** $2x^2 + 3x \leqslant 0$

2 Find the set of values of x for which:

a $x^2 < 10 - 3x$ **b** $11 < x^2 + 10$

c $x(3 - 2x) > 1$ **d** $x(x + 11) < 3(1 - x^2)$

3 Use set notation to describe the set of values of x for which:

a $x^2 - 7x + 10 < 0$ and $3x + 5 < 17$

b $x^2 - x - 6 > 0$ and $10 - 2x < 5$

c $4x^2 - 3x - 1 < 0$ and $4(x + 2) < 15 - (x + 7)$

d $2x^2 - x - 1 < 0$ and $14 < 3x - 2$

e $x^2 - x - 12 > 0$ and $3x + 17 > 2$

f $x^2 - 2x - 3 < 0$ and $x^2 - 3x + 2 > 0$

4 Given that $x \neq 0$, find the set of values of x for which:

a $\frac{2}{x} < 1$

b $5 > \frac{4}{x}$

c $\frac{1}{x} + 3 > 2$

d $6 + \frac{5}{x} > \frac{8}{x}$

e $25 > \frac{1}{x^2}$

f $\frac{6}{x^2} + \frac{7}{x} \leqslant 3$

5 **a** Find the range of values of k for which the equation $x^2 - kx + (k + 3) = 0$ has no real roots.

b Find the range of values of p for which the roots of the equation $px^2 + px - 2 = 0$ are real.

Hint The quadratic equation $ax^2 + bx + c = 0$ has real roots if $b^2 - 4ac \geqslant 0$. ← Section 2.5

6 Find the set of values of x for which $x^2 - 5x - 14 > 0$. **(4 marks)**

7 Find the set of values of x for which

a $2(3x - 1) < 4 - 3x$ **(2 marks)**

b $2x^2 - 5x - 3 < 0$ **(4 marks)**

c **both** $2(3x - 1) < 4 - 3x$ **and** $2x^2 - 5x - 3 < 0$. **(2 marks)**

8 Given that $x \neq 3$, find the set of values for which $\frac{5}{x - 3} < 2$. **(6 marks)**

Problem-solving

Multiply both sides of the inequality by $(x - 3)^2$.

9 The equation $kx^2 - 2kx + 3 = 0$, where k is a constant, has no real roots. Prove that k satisfies the inequality $0 \leqslant k < 3$. **(4 marks)**